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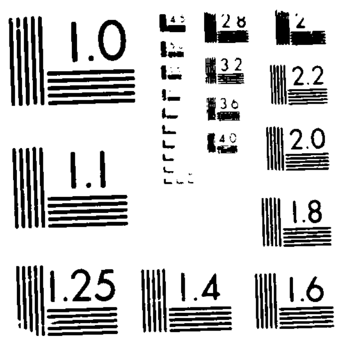

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ALGORITHMIC GENERATION OF FEASIBLE SYSTEM ARCHITECTURES*

by

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ABSTRACT

A methodology is presented for generating decisionmaking organizational structures that satisfy some given requirements. Allowable interactions between organizational members are first defined and constraints are introduced. To find the set of structures that satisfy the requirements, a methodology has been designed that reduces the computational complexity of the problem and makes it tractable. The set of structures is delimitedby its maximal and minimal elements and a technique is given to generate the entire set from its boundaries. Simple paths are introduced as the incremental unit leading from an organization to its neighboring ones. Lattice theory is used throughout the methodology as the underlying analytical tool.

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1. INTRODUCTION

Information processing and decisionmaking organizations have been modeled and analyzed using Petri Nets [1], [2], [3], [4]. The organizational forms that can be modeled by Petri Nets is only limited by the imagination of the designer. To make the problem tractable, a framework needs to be defined that will restrict the class of organizational structures under consideration.

The first step of the approach consists of defining the general framework within which organizational forms will be built. This framework will define the allowable structure of interactions among decisionmakers. It uses, as a starting point, the four stage representation of the single interacting decisionmaking [5]. This general model will condition the scope of the entire design methodology. It needs to be enough to reflect relatively sophisticated situations, without getting mathematically out of hand.

In the second step, the organization designer will restrict the class of organizational forms by imposing constraints on organizations. In addition to the designer's requirements, the organizational structures to be generated must satisfy a set of structural constraints reflecting some generic properties.

The last step consists in finding the set of all organizations that satisfy both the designer's and the structural constraints. Results from lattice theory are used to characterize this set thanks to its minimal and maximal elements. Lattice theory is also used to investigate the interval structures of the set.

The overall procedure has been implemented on a personal computer. It allows the organization designer to go step by step through the entire design methodology.

ORGANIZATIONAL CLASSES

The first step of a methodology for designing decisionmaking organizations is the modeling of a single decisionmaker. A somewhat simplified version of the four stage model is reproduced in Figure 1. A stage is represented by a transition. The decisionmaker receives a signal x - from the external environment (2) or from another organization member (1). The situation assessment (SA) stage contains algorithms that process the incoming signal to obtain the assessed situation z. The assessed situation z may be reported to other members: a copy of it is communicated via one or more interactional places as represented in Figure 1. Concurrently, the decisionmaker can receive a signal z'' from another part of the organization; z'' and z are then merged together in the information fusion (IF) stage to produce z'. The possibility of receiving commands from other organization members is reflected in the variable v'. The command interpretation (CI) stage combines z' and v' to produce the variable v that contains z' and the appropriate strategy to use in the response selection (RS) stage. Finally, the RS stage contains algorithms that produce the output y.

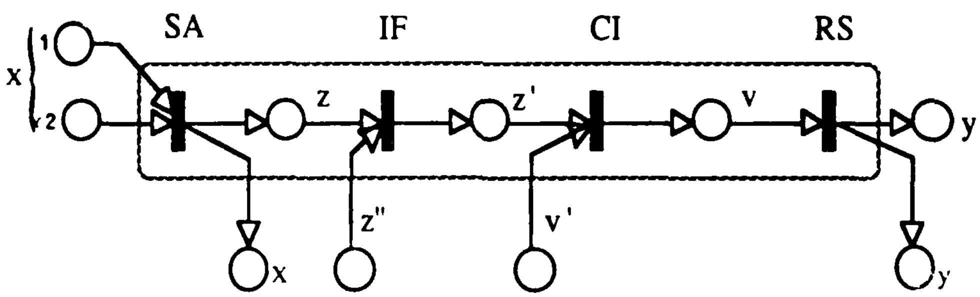

Figure 1. Four stage model of a decisionmaker

This model shows explicitly at which stage a decisionmaker can interact either with the external environment or with other organization members. A decisionmaker need not have all four stages. If any two stages are present, however, their intermediate stages must also be present. The set of all allowable interactions is represented in Figure 2. Some possible links have been ruled out to reduce the dimensionality of the design problem, while being consistent with the conventions adopted for this model. Links from DM^i to DM^j only have been represented. Symmetrical links from DM^j to DM^i are of course valid interactions. However, as will be described in detail, not all interactions are allowed to occur at the same time, i.e., to appear in the same net model.

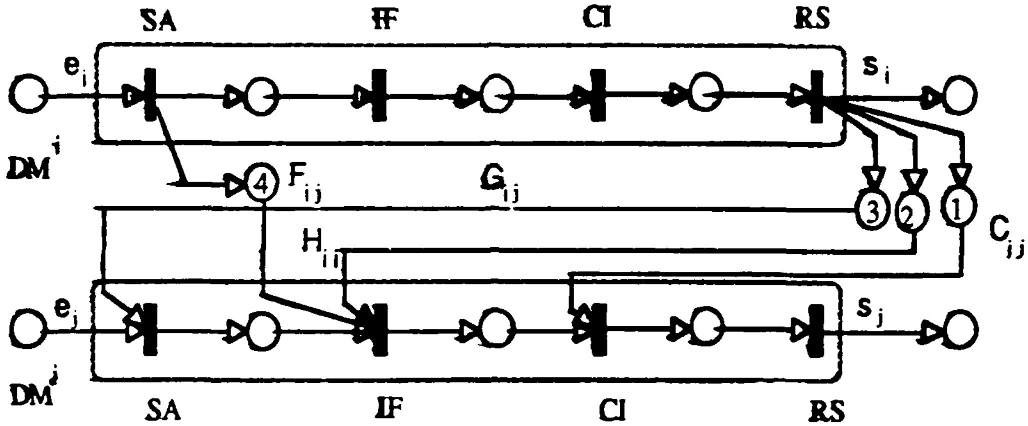

Figure 2. Allowable interactions

There are four possible links from a decisionmaker to another one and the maximum number of links, k_{max}, in a n-decisionmaker organization is therefore

$$k_{max} = 4n^2 - 2n. \tag{1}$$

Mathematical Representation of Interactions

The mathematical representation of interactions between decisionmakers is based on the connector labels $e_i, s_i, F_{ij}, G_{ij}, H_{ij}, C_{ij}$ of Figure 2; they are integer variables taking values in {0,1} where 1 indicates that the corresponding directed link is actually present in the organization, while 0 reflects the absence of the link. These variables are aggregated into two vectors $\underline{e}$ and $\underline{s}$, and four matrices F, G, H, and C. The interaction structure of an n-decisonmaker organization will therefore be represented by the following six arrays.

Two n × 1 vectors $\underline{e}$ and $\underline{s}$, representing the interactions between the external environment and the organization:

$$\underline{e} \equiv [e_i]; \qquad \underline{s} \equiv [s_i]; \qquad i = 1,2,\ldots,n. \tag{2}$$

Four n × n matrices F, G, H, C representing the interactions between decisionmakers inside the organization:

$$F \equiv [F_{ij}]\ ;\ G \equiv [G_{ij}]\ ;\ H \equiv [H_{ij}]\ ;\ C \equiv [C_{ij}]$$

$$i = 1,2,\ldots,n \qquad \text{and} \quad j = 1,2,\ldots,n. \tag{3}$$

The six-tuple $\{\underline{e},\underline{s},F,G,H,C\}$ will be called a **Well Defined Net** (WDN) of dimension n, where n is the number of decisionmakers in the organization. The set of all Well Defined Nets of dimension n will be denoted Ψ^n. It is clear that Ψ^n is isomorphic to the set $\{0,1\}^{k_{max}}$, where k_{max} is given by eq.(1). The cardinality of Ψ^n is therefore

$$2^{k_{max}} = 2^{4n^2 - 2n}. \tag{4}$$

The notion of a **subnet** of a WDN can be defined as follows. Let $\Pi=\{\underline{e}, \underline{s}, F, G, H, C\}$ and $\Pi'=\{\underline{e}', \underline{s}', F', G', H', C'\}$ be two WDNs. The WDN Π' is a subnet of Π if and only if

$$\underline{e}' \leq \underline{e} \qquad F' \leq F \qquad G' \leq G$$
$$\underline{s}' \leq \underline{s} \qquad H' \leq H \qquad C' \leq C$$

where the inequality between arrays is interpreted element by element.

In other words, Π' is a subnet of Π if any interaction in Π', i.e. a 1 in any of the arrays $\underline{e}',\underline{s}',F',G',H',C'$, is also an interaction in Π. The union of two subnets Π_1 and Π_2 of a WDN Π, is a new net that contains all the interactions that appear in either Π_1 or Π_2 or both.

The notion of subnet introduced earlier defines an order on the set ψ^n: we will write $\Pi' \leq \Pi$ if and only if Π' is a subnet of Π. The set Ψ^n with the relation "$\leq$" is a partially ordered set. It can be shown [6], that Ψ^n satisfies the Jordan-Dedekind chain condition [7]. Moreover, two WDNs have always a least upper bound (l.u.b.) and a greatest lower bound (g.l.b.) within the set Ψ^n [6]. Consequently Ψ^n is a lattice.

DECISIONMAKING ORGANIZATIONS

The notion of Well Defined Net (WDN) has been introduced to characterize the class of organizations under consideration; however, each WDN is not a valid organizational structure. The **structural constraints** define what kinds of combinations of interactions need to be ruled out. **User-defined constraints** are used to allow the designer to introduce specific structural characteristics appropriate to the particular design problem. Four different structural constraints are formulated that apply to all organizational structures being considered.

- (R_1) A directed path should exist from the source to every node of the structure and from every node to the sink.
- (R_2) The structure should have no loop, i.e., the organizational structures are acyclical.
- (R_3) There can be at most one link from the RS stage of a DM to each one of the other DMs, i.e., for each i and j, only one element of the triplet $\{G_{ij}, H_{ij}, C_{ij}\}$ can be nonzero.
- (R_4) Information fusion can take place only at the IF and CI stages. Consequently, the SA stage of each DM can have only one input.

The set of structural constraints is defined as $R_s = \{R_1, R_2, R_3, R_4\}$.

The constraint R_1 defines connectivity as it pertains to this problem. It eliminates structures that do not represent a single integrated organization and ensures that the flow of information is continuous within an organization. Note that constraint R_1 ensures that the Petri Net representing an organization whose source and sink have been merged together, is strongly connected. Constraint R_2 allows acyclical organizations only. Constraint R_3 states that a decisionmaker can send its output of the RS stage to another given decisionmaker only once. It does indeed not make much sense to send the same output to the same decisionmaker at several different stages. Constraint R_4 prevents a decisionmaker to receive more than one input at the SA stage. The logic behind this limitation is that information cannot be merged at the SA stage. The IF stage has been specifically introduced to perform such a fusion. Note that Figure 2 does not fulfill (R_3) and (R_4).

The organization designer implements user-defined constraints by placing the appropriate 0's and 1's in the arrays $\{\underline{e}, \underline{s}, F, G, H, C\}$ defining a WDN. The other elements will remain unspecified and will constitute the degrees of freedom of the design. The set of user-defined constraints will be denoted R_u, while the complete set of constraints will be denoted R.

A WDN that fulfills the set of user-defined constraints R_u will be called an **Admissible Organizational Form** (AOF). The set of all AOFs will be denoted $\Phi(R_u)$.

An AOF that fulfills the set of constraints R_s will be called a **Feasible Organization** (FO). Note that a Feasible Organization is a WDN that fulfills the complete set of constraints R. The set of all Feasible Organizations will be denoted $\Phi(R)$. Trivially, the following inclusions hold:

$$\Psi^n \supset \Phi(R_u) \supset \Phi(R)$$

Structure of the set $\Phi(R_u)$

Let us define the *Universal Net*, $\Omega(R_u)$, associated with the constraints R_u as the WDN obtained by replacing all undetermined elements of the arrays $\underline{e}$, $\underline{s}$, F, G, H, and C by 1. Similarly, the *Kernel Net*, $\omega(R_u)$, will be the WDN obtained by replacing the some undetermined elements by 0.

It has been shown [6] that the set $\Phi(R_u)$ is characterized by the following equality

$$\Phi(R_u) = \{\Pi \in \Psi^n / \omega(R_u) \leq \Pi \leq \Omega(R_u)\}$$

As a corollary, $\Phi(R_u)$ is a sublattice of Ψ^n. The goal is to find a similar characterization for the set $\Phi(R)$.

Characterization of the set $\Phi(R)$

If the methodology presented in this paper is to have any practical use, its needs to yield a reasonable number of candidate feasible organizations that can then be analyzed by the designer. Unfortunatelly, in the general case, the cardinality of the set $\Phi(R)$ can be huge, which poses both a computational and methodological problem. This section proposes on approach to cope with this complexity. In the first step, the boundaries of the set $\Phi(R)$ are defined. In the next step, the inner part of the set is investigated. The notion of simple path is introduces as the incremental unit leading from a FO to a neighboring one. This yields a procedure for building the entire set $\Phi(R)$ from its boundaries. Lastly, a few mathematical properties of $\Phi(R)$ are discussed. The ultimate goal would be to divide the set $\Phi(R)$ into few categories and to select for the designer a representative among each category. This section is a first step along those lines.

Minimally and Maximally connected organizations

A maximal element of the set $\Phi(R)$ of all Feasible Organizations will be called a **Maximally Connected Organization** (MAXO). Similarly, a minimal element of $\Phi(R)$ will be called a **Minimally Connected Organization** (MINO). The set of all MAXOs (resp. MINOs) will be denoted $\Phi_{max}(R)$ (resp. $\Phi_{min}(R)$).

Maximally and minimally connected organizations can be interpreted as follows. A MAXO is a WDN such that it is not possible to add a single link without violating the set of constraints R (i.e. without crossing the boundaries of the subset $\Phi(R)$). Similarly, a MINO is a WDN such that it is not possible to remove a single link without violating the set of constraints R. The following proposition is a direct consequence of the definition of maximal and minimal elements.

Proposition 1

For any given Feasible Organization Π, there is at least one MINO Π_{min} and at least one MAXO Π_{max} such that $\Pi_{min} \leq \Pi \leq \Pi_{max}$. Alternatively,

$$\{\Pi \in \Psi^n \mid \exists(\Pi_{min},\Pi_{max}) \in \Phi_{min}(R) \times \Phi_{max}(R) \Pi_{min} \leq \Pi \leq \Pi_{max}\} \supset \Phi(R)$$

Note that the previous inclusion is not an equality in the general case. There is indeed no guarantee that a WDN located between a MAXO and a MINO will fulfill the constraints R, since such a net need not be connected. To address this problem, the concept of a simple path is used.

Let Π be a WDN that satisfies constraint R_1. A **simple path** of Π is a directed line from the source to the sink. Petri Nets are used at this stage to find single paths [1], [8].

Single paths of a WDN are themselves WDNs. Let us denote by $Sp(R_u)$ the set of all simple paths of the Universal Net $\Omega(R_u)$. We will write

$$Sp(R_u) = \{sp_1, \ldots, sp_r\},$$

where the sp_i $(1 \leq i \leq r)$ are WDNs satisfying $sp_i \leq \Omega(R_u)$.

If the cardinality of $Sp(R_u)$ is r, we can write $Sp(R_u) = \{sp_i, 1 \leq i \leq r\}$. Since simple paths are WDNs, the set $Sp(R_u)$ is included in the set of all WDNs, Ψ^n. We will denote by $USp(R_u)$ the set of all possible unions of elements of $Sp(R_u)$, augmented with the null element φ of Ψ^n, i.e., the WDN with all elements identically equal to zero.

$$USp(R_u)=\{\Pi\in \Psi^n | \exists(sp_{i1},...sp_{1q})\in Sp(Ru)^q$$

$$\Pi=sp_{i1}\cup...\cup sp_{iq}\}\cup\{\varphi\}$$

$USp(R_u)$ is the set of all combinations of simple paths of the Universal Net $\Omega(R_u)$. The union of two elements of $USp(R_u)$ will be the WDN composed of all the simple paths included in **either one** of the two considered elements. Proposition 2 justifies the introduction of the set $USp(R_u)$.

Proposition 2

Every WDN, element of the set $USp(R_u)$, satisfies the connectivity constraint R_1.

Reciprocally, a Feasible Organizational Form that fulfills the constraint R_1 is an element of $USp(R_u)$. In formal language:

$$\{\Pi \in \Psi^n \mid R_1[\Pi] = 1\} \supset USp(R_u) \supset \{\Pi \in \Phi(R_u) \mid R_1[\Pi] = 1\}$$

$R_1[\Pi] = 1$ means that Π satisfies the constraint R_1. It is easy to see that the set $USp(R_u)$ is a sublattice of Ψ^n.

We are now ready to state the following proposition characterizing the set $\Phi(R)$ of all feasible organizations.

Proposition 3

Let Π be a WDN of dimension n. Π will be a Feasible Organization if and only if

- Π is a union of simple paths of the Universal Net $\Omega(Ru)$, i.e., $\Pi \in USp(R_u)$.
- Π is bounded by at least one MINO and one MAXO.

Formally:

$$\Phi(R) = \{\Pi \in USp(R_u) \mid \exists(\Pi_{min}, \Pi_{max}) \in \Phi_{min}(R) \times \Phi_{max}(R) \ \ \Pi_{min} \leq \Pi \leq \Pi_{max}\}$$

Proposition 3 gives a characterization of the set $\Phi(R)$ just like Proposition 2 gives a characterization of the set $\Phi(R_u)$. While Ψ^n is used in the equality characterizing $\Phi(R_u)$, $USp(R_u)$ is used to characterize $\Phi(R)$. In the former case, the link is the incremental unit leading from a WDN to its immediate superordinate, while in the latter the simple path plays the role of the building unit. In generating organizational structures with simple paths, the connectivity constraint R_1 is automatically satisfied.

Structure of the set $\Phi(R)$

In the general case $\Phi(R)$ will not be a lattice as the following porposition shows.

Proposition 4

The set $\Phi(R)$ is a lattice if and only if $\Phi(R)$ has exactly one MINO and one MAXO.

This proposition is a rather negative result since in most cases there will be several MAXOs and MINOs. To gain deeper insight into the structure of $\Phi(R)$ the notion of minimal decomposition is introduced.

Let Π be an element of $\Phi(R)$ and let $USp(\Pi)$ be the lattice polynominal generated by all the single paths of Π. A *minimal decomposition* of Π will be a family of single paths of Π that constitutes a *minimal length chain* [7] leading from the null element ϕ to Π in the lattice $USp(\Pi)$.

Formally, a minimal decomposition of Π is a family $A=\{sp_{i1},...,sp_{is}\}$ of single paths Π, satisfying the following conditions:

- $\Pi = sp_{i1} \cup ... \cup sp_{is}$
- $\phi < sp_{i1} < sp_{i1} \cup sp_{i2} < ... < sp_{i1} \cup ... \cup sp_{is} = \Pi$
- The length of any chain form ϕ to Π is at least s.

The operator "$\cup$" denotes the join operator. Note that Π may have several minimal decompositions. The number s is however invariant: it will be called the complexity of Π and denoted $C(\Pi)$.

Intuitively, $C(\Pi)$ is the minimal number of single paths necessary to built Π. The following proposition shows how the complexity behaves with the join ("$\cup$") and the meet ("$\cap$") operators:

Proposition 5

Let Π and Π' be too FOs, elements of $\Phi(R)$. The following inequality holds:

$$C(\Pi \cup \Pi') + C(\Pi \cap \Pi') \leq C(\Pi) + C(\Pi')$$

An inductive method is used to prove this proposition [6].

CONCLUSIONS

In this paper, a methodology is presented for generating organizational architectures that satisfy some generic structural properties, as well as more specific designer's requirements. An analytical framework is developed to formulate first and then analyze the problem.

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